A Note on Surplus Content

In various recent publications, including Yablo (2014, 2016), Stephen Yablo appeals to the notion of surplus content. Why, for instance, does

1 The number of words in the title of this paper is five;

seem unproblematic, while its straightforward consequence

2 There is at least one number;

is ontologically controversial? Yablo's diagnosis: Because when uttering (1) ordinary speakers assert only (1)'s *surplus content* with respect to (2). They only assert

3 *If* there is at least one number, the number of words in the title of this paper is five;

on a certain (non-truthfunctional) reading of 'if/then'. Since (3) does not entail (2), but rather expresses *what is left of* (1) when you subtract this consequence, the unsettling drop in plausibility between what is asserted by sentence (1) and its straightforward consequence (2) can be explained.

When Yablo introduces surplus content he does so by appeal to the idea of mathematical addition and subtraction. The content of (the pertinent reading of) (3) is what you get when you *subtract* the content of (2) from the content of (1). Correlatively, if you *add* the content of (2) to the content of (3), you get the full content of (1). Since addition and subtraction are familiar operations on the real numbers, this seems to give us some handle on the operation Yablo asks us to perform on contents.

However, there is a straightforward concern about this appearance. In mathematics, subtraction is typically defined as the addition of the inverse: '$a - b$' is short for '$a + (-b)$'. The inverse (with respect to addition) of a number b in turn is defined as that number which added to b yields the neutral element with respect to addition. The neutral element, finally, is that number which added to any number a yields a. If we are to make literal sense of Yablo's suggestion that surplus content is what remains when we subtract, we should be able to say what sort of operation content-addition is (including what set of things it is an operation on), what the neutral element with respect to content-addition is, and for which contents there are inverses under that operation. This is not a trivial matter. In fact, on a natural choice for the set of contents, these questions have answers that are disastrous for the envisaged applications of the idea. It would, thus, seem that Yablo should either specify his preferred alternative construction or refrain from appealing to the familiar notions of mathematical addition and subtraction in explaining what surplus content is supposed to be.

Here is the straightforward construction. Think of the set of contents on which the operation of content-addition is defined as the set of Lewis-propositions: sets of possible worlds at which the relevant contents are true (including the empty set, and the set of all worlds). Content-addition may simply be taken to be conjunction, i.e. intersection of Lewis-propositions. You content-add two propositions by conjoining them. Since intersection of Lewis-propositions always yields a Lewis-proposition, content-addition will be an operation on the set of Lewis-propositions. The neutral element with respect to content-addition on the set of Lewis-propositions is the set of all worlds: whenever you content-add the set of all worlds to any Lewis-proposition, the result is that Lewis-proposition again, since intersecting any set of worlds A with the universal set of worlds yields A. Nothing less than the set of all worlds will do, since if an arbitrary world w is not included in the neutral element, the intersection of the proposition $\{w\}$ with the alleged neutral element yields the empty set, rather than $\{w\}$. Given this natural choice for content-addition and neutral element, it turns out that *only one* Lewis proposition has an inverse under content-addition, namely the trivial/necessary Lewis-proposition (the inverse is the necessary proposition itself): For, only the set of all worlds can be *intersected* with any set of worlds to yield the neutral element. Correlatively, content-subtraction is *only* defined in our construction for the trivial proposition: you can subtract the trivial proposition from any proposition A, and you will get back A. But this will not help Yablo in explaining how highly non-trivial content can be subtracted from a proposition such as the one expressed by (1). Nor will it help Yablo in explaining how what remains when we subtract is less committal than what we started out with.

Of course, Lewis-propositions are much too coarse-grained to play the role of contents in Yablo's theory. But the problem does nor arise because of the coarseness of grain of Lewis-propositions: Being able to make finer distinctions allows us to have more than one necessary proposition, and, thus, more than one proposition that has an inverse under the present construction. But it does not help with providing non-necessary propositions with inverses. In particular, Yablo-contents are Lewis-propositions enriched with other information (their truth-makers). So, it would seem that talk of subtraction should make sense for the more coarse-grained siblings of Yablo-contents, if it makes sense for Yablo-contents. And this is cast into doubt by the considerations in the previous paragraph.

How important is it for Yablo to provide all kinds of propositions with inverses? The choice of example might make us suspect that it is enough for Yablo's purposes that the necessary Lewis-proposition has one (namely itself). For, orthodoxy might be inclined to think, if there are numbers, this is necessary, so it might be enough to be able to subtract necessary Lewis-propositions (the action would then presumably be in the enrichment that make Yablo-contents more fine-grained than their Lewisian bases). But, of course, this is a big *if*, and one that the nominalist should not be comfortable in relying on! Also, the number case is only one of many to which Yablo wishes to apply the idea of content-subtraction: ‘I think’ and ‘There are thinkers’, ‘Here is a hand’ and ‘There are material objects’ and ‘This is a zebra’ and ‘This is a cleverly disguised mule’, for instance. None of the contents that would need to be subtracted here are even prima facie candidates for being built on necessary Lewis-propositions.

The concern raised here suggests that the analogy with mathematical subtraction is underdeveloped and potentially misleading. Perhaps, there are more promising ways of developing an account of surplus content available for Yablo’s purposes. Thus, Yablo (2016) also gives four principles that are meant to govern surplus content, thus hinting at an implicit definition via these principles. Humberstone (2011) and Fine (201?a) propose semantic accounts of the notion. However, as Fine (201?b) points out, Yablo’s principles presuppose an underlying account of the truth-makers for the material conditional that remains unspecified (while the two accounts already on the market yield Yablo-unfriendly results). On the other hand, both Humberstone and Fine stress that their own variants of content subtraction do not apply to arbitrary propositions. For instance, on Fine’s account, $A \sim B$[1] only exists if the states that potentially make A and B true are related in the right way. Very roughly, each (exact) truth-maker for A has to be the fusion of some B-independent part c and a B-ish part b. Similarly, Humberstone (2011: ch. 5.2) discusses the suggestion that for $A \sim B$ to be defined, A has to be understandable as a conjunction of some B-independent conjunct with a B-ish conjunct. This is why, for instance, Wittgenstein’s case of subtracting the proposition that my arm goes up from the proposition that I raise my arm, or that of subtracting the proposition that the ball is coloured from the proposition that the ball is red strike us as barely intelligible: truth-makers for the proposition that I raise my arm aren’t plausibly conceived of as fusions of two independent states of affairs one of which concerns my arm’s going up, nor is the proposition that the ball is red a conjunction of its being coloured and X, where X is independent of the ball’s being coloured.[2] Yet it is at the very least unclear whether all the cases Yablo intends his account to apply to are of the favourable kind. In particular, one of the cases Yablo (2016: §1) cites concerns number talk applied to a phenomenon of pure mathematics itself: ‘The number of even primes is 1’. According to Yablo, what is asserted is the surplus content that sentence has in addition to the proposition that there are numbers, i.e. *the number of even primes is 1 ~ there are numbers*. But it is very doubtful that the proposition literally expressed by the sentence can be conceptualized as a conjunction of the proposition that there are numbers with some other proposition *that is independent of there being numbers*. Analogously, it is very doubtful that the proposition literally expressed by the sentence has potential truth-makers that are fusions of number-involving states of affairs with number-independent states. If this is correct, the alternative accounts of surplus content are unavailable for Yablo’s purposes as well.

[1] ‘$A \sim B$’ is short for ‘the result of content-subtracting B from A’.

[2] For a similar point regarding his ground-theoretic reconstruction of Yablo’s ideas see Rosen (2016: §III).

References

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